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EXPERIMENTAL MODERN PHYSICS:
WHY DO WE NEED NEW MATHEMATICS?

Summary

One hundred years or so, it was believed that the knowledge on Physics was complete and only tiny improvements are needed. Soon, Quantum Physics was discovered and the XIX century mathematical apparatus turned out useful in calculations of hydrogen-atom emission spectra and in designing pharmaceutical products, now.

It seems that we may face a similar revolution in the near future. A lot of experimental evidence in Physics does not find clear theoretical explanations; numerous come to a total surprise. We mention here: in Solid State Physics high-T superconductors, in Elementary Particle Theory – the wide (and apparently) disordered range of quark masses and the CP broken symmetry; in Cosmology – uknown solution of the General Relativity and the problem of dark mass and energy. With a high probability new ideas from Mathematics are needed for Physics.

1. Historical outline

The starting point of Quantum Physics is usually identified with the lecture of Planck at Berlin meeting of the Deutschen Physikalischen Gesellschaft on Dec. 14th, 1900, where he put the hypothesis of quanta as the portion of the radiative energy. In common understanding it appears to be a kind of ad hoc hypothesis invented by Planck to avoid the problem with the black-body radiation density spectra in the high-frequency range (so called the UV catastrophy). However, a precise inspection of Planck's papers proves different.

Planck, searching for the unification between Maxwell-Boltzmann thermodynamics and Maxwell electrodynamics, produced several formulae which had predicted pretty well the experimentally measured intensity of radiation. However, at about mid October 1900 new measurements differed from Planck's formulae, even if with the difference almost impossible to notice. Planck, in several weeks of hard work found a new formula, approximating better the experimental data and, using concepts of entropy and energy, invented Quantum Physics.

Einstein received the Nobel prize for his 1905 explanation of the photoelectrical effect. Once again, the experimental evidence was only faint: the measurement by Paul Lenard who, not having intense sources of light (and with a variable frequency characteristics) used the carbon and zinc-arc lamps. The two retarding-field spectra differed almost null but their explanation required the hypothesis that the light is not only emitted by quantified harmonic oscillators like it was for Planck, but that it also brings quantized energy portions.

Abraham Michelson's experiment on the light velocity, done in Potsdam already in 1881, showed that this velocity did not depend on Earths motion but the evidence was almost within the experimental uncertainty. The experiment awaited 25 years to be explained by Einstein, and brought also the new understanding of the energy and mass $E = mc^2$.

Several experiments, in cosmology, elementary particle, solid state Physics require new theoretical ideas. We outline some of them below. However, we start from the own field, i.e. Atomic Physics and electronic optics.

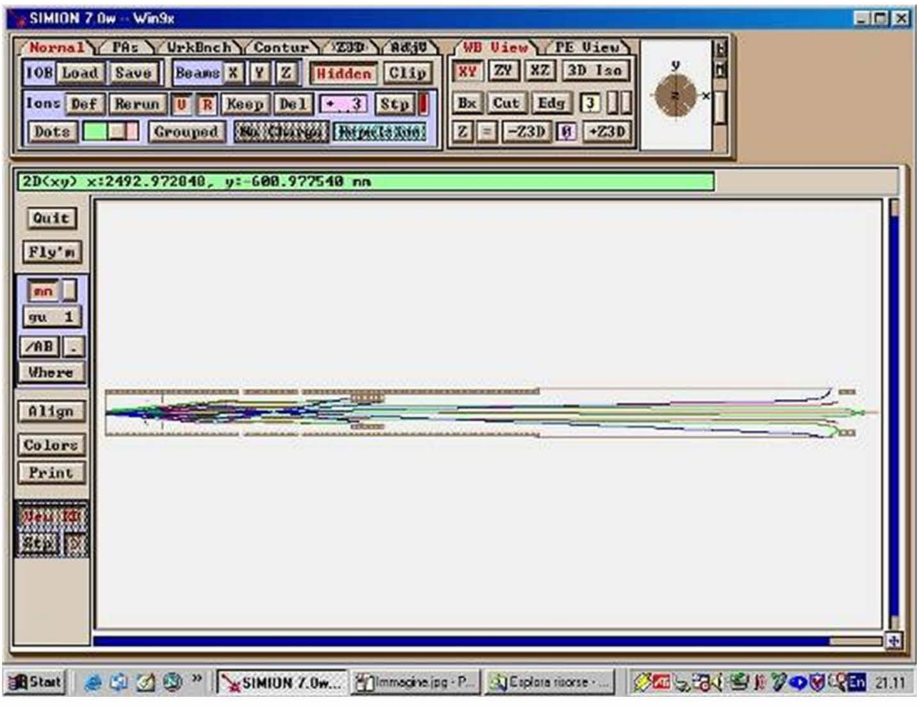

Fig. 1: Modelling trajectories in low-energy positron-atom scattering apparatus [1]. Modelling uses the SIMION numerical package, but does not include the magnetic focusing. As a result, modeling underestimated the real positron flux obtained in the experiment [1].

2. Electron (and positron) optics

Roman Ingarden in his PhD thesis in 1957 considered the use of new geometry, now called Randers-Ingarden geometry, for modeling motion of electrons in an electron microscope. Nowadays, numerical packets allow to model even complex electron optics, but these programs are not perfect and experimental setting of apparates need some laboratory skills. We give an example of the recent apparatus for positron-atom scattering from Trento University, in which the setting used [1] differed significantly from those projected: we have "inverted" the optics operation from an accelerating ($200 \to 2000\,\mathrm{eV}$) to the decelerating ($200 \to 20\,\mathrm{eV}$) mode.

This in principle is possible, but as the initial energy of positron is 2 eV with the energy spread of about 1 eV, the numerical modeling, see Fig. 1, gave the very low beam intensity, say of positrons per second. In spite of this, the convoluted use of the low-intensity, longitudinal guiding magnetic field allowed us to work with as high as 10-100 positrons per second counting rate and obtain some intriguing, new physical results [1].

The numerical packets used, like SIMION, based on Poisson equation (in "normal" 3D space), hardly allow to include effects of the magnetic field, which performs additional focusing, but with a-quasi resonant conditions (an integer number of positron spiraling inside the scattering cell). We question, if the use of more sophisticated approach, say Randers-Ingarden geometries would facilitate modelling of the positron optics in this practical case.

3. Solid State Physics

The quantum Hall effects was discovered in 1980 by Klaus von Klitzing. Working with high magnetic fields he found that at low temperatures the Hall voltage (perpendicular to the direction of the conduction current and arising from curving electron paths in the magnetic field) changes in a step-like manner. The successive values of the Hall conductance are integer multiples of the constant $e^2/h = 1/(25812.807572\,\Omega)$. On this way, the quantum Hall effect allows to measure with a high precision fundamental constants of nature irrespective of the sample imperfections. It was shown only recently that quantization of the Hall conductance results from topological considerations on the Hamiltonian: under great deformations of Hamiltonian the curvature of the Hall conductance passes from one topological Chern number to another [2].

Also recently, new phenomena have been discovered for a class of materials with a high spin-orbit coupling, in which the internal magnetic field substitutes effects of the external magnetic field. In HgTe-CdTe quantum wells changing the thickness of the well induces the transition from an insulating state to a phase exhibiting quantum spin Hall effect. It has been recently shown that this change results from a topological quantum phase transition [3].

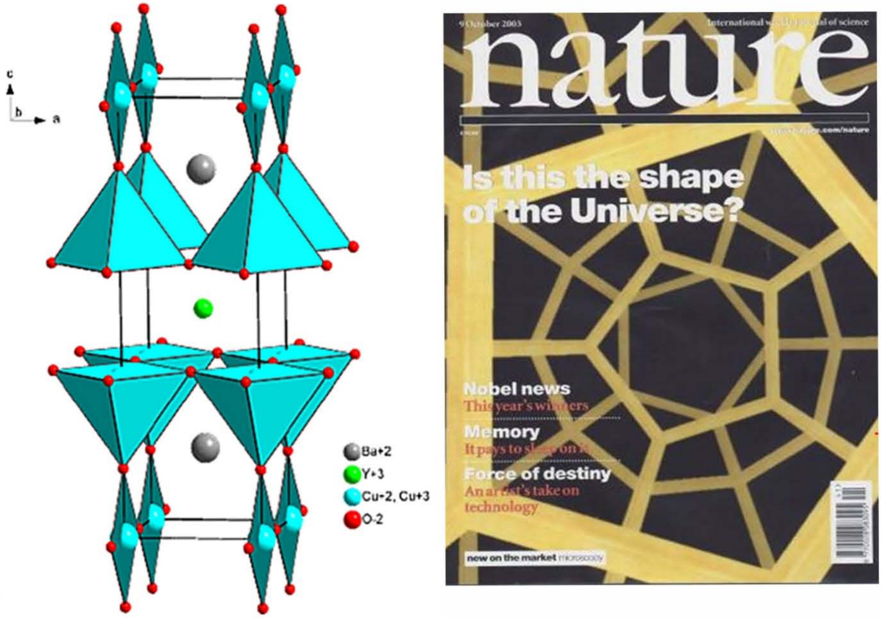

Fig. 2: Left panel: crystalographic structure of YBCO superconductors. Right panel: A possible shape of the Universe deduced from microwave-background radiation.

Another class of new materials with predicted unusual properties is $Bi_{1-x}Sb_x$ [4]. In the normal state it is an insulator but due to topological considerations becomes a metal on the surface. This is again, as in the case of quantum Hall effect, due to the fact that the Hamiltonian describing the surface states is invariant to small perturbations and defined by topological Chen numbers.

Quoting Shou-Cheng Zhang [5]: "Topological states of quantum matter now offer a new laboratory to test some of the most profound ideas in mathematics and physics. In 2007, the theoretical prediction and experimental observation of the quantum spin Hall state – a topological insulator in two dimensions – in HgTe quantum wells was highlighted as one of the top ten breakthroughs among all sciences."

Superconductivity was discovered experimentally in 1911 by Kamerlingh Onnes (in Hg at 4.2 K). Surprisingly, good conductors like Cu or Ag do not become superconductors. Only in 1957 a theoretical explanation came by Bardeen, Cooper and Schrieffer. The superconductivity is described by the Bose-Einstein statistics, the same as used by Planck for photons. So called 2nd type superconductors were discovered in Nb_3Sb in 1930 and explained theoretically by Abrikosov in 1954 assuming quantization of the magnetic flux in vortices. However, this theory did not predict superconductivity in copper oxides, at as high temperatures as 77 K and above, discovered in 1986. All new high-T superconducting materials ($YBa_2Cu_3O_7$ – 92 K, $HgBa_2Ca_2Cu_3O_8$ – 135 K etc.) posses complex, layered crystallographic structures, see Fig. 2.

In spite of several decades passed from the experimental discovery of copper-oxide superconductors, we still lack a plausible explanation. Some of the existing theories evocate quite fantastic explanations like time reversibility and quantum fluctuations rising with the temperature decrease. We note that in Hall effect the transport phenomena occur only in two dimensions and samples are uniform. The high-T superconductors have much more complicated structures. The lack of proved explanations evocate questions on the new Mathematics needed for the high-T superconductivity theory – fractal geometries or some new topological arguments (?).

4. Elementary particles

The 2008 Nobel prize was assigned for the prediction in 1974 of the third generation of quarks by Kobayashi and Maskawa. These quarks (and earlier the third lepton, "tau") were soon discovered in experiments with huge accelerators at Fermilab. Now we know the masses of quarks with a pretty high accuracy (except the two most common, *up* and *down*). They scale in a mysterious way: 3, 6, 1500, 105, 170.000, 4700 (in MeV/c^2 units). Why? We do not know. Quoting prof. L. Pitaevsky: "There are some people saying that if it were different, we would not be here, able to think about it."

Another open question in the field of elementary particles is the mass of the neutrino – a photon-like companion of leptons, born in weak-decay reactions. As far as electron and muon neutrinos were detected already at mid of XX century, the question of their mass is still open. Furthermore, for almost 50 years, a strong discrepancy between models of nuclear reaction in the Sun and the detected neutrino flux from the Sun was not solved. Only in 2002 the detection of both electron and muon neutrinos from Sun showed that these two forms can transform each into another [6].

In 2003, experiments from Kamioka Laboratory gave another sensational notice [7]: anti-neutrinos from Japanese reactors dissapeared in a mysterious way. The only explanation was that electronic neutrinos changed their flavour in flight, becoming muon neutrinos. So neutrinos posses mass! We still do not know their values but the difference in mass between ν_e and ν_μ is very small: $\Delta m = 6.9 \times 10^{-5}\,\mathrm{eV}^2$. Not only we are not able to predict masses of neutrinos but we are not able even to classify clearly them into schemes of other, mass-possessing elementary particles.

The most promising hypothesis in the recent decades seemed the string theory. But it is still not able to predict the lifetime of proton or masses of quarks. G. Esaterbrook says: "Although string theory, like any other scientific theory, is falsifiable in principle, critics maintain that it is unfalsifiable for the foreseeable future, and so should not be called science" [8].

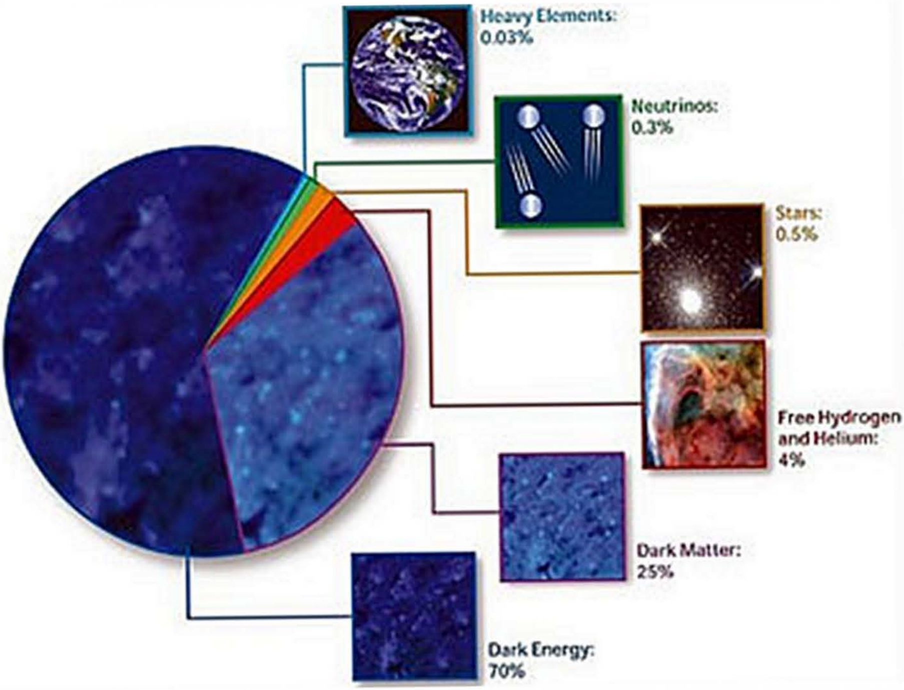

Fig. 3: Cosmological puzzle: the narrow coloured slice is the Universe we detect with all known-to-us methods. The rest is present, but simple invisible to us. Source: "Nature" [9].

5. Cosmology

The question of steady-state or gravitationally shrinking Universe was not obvious for Einstein when he presented the general theory of relativity. On four Thursday meeting of the Preussische Akademie der Wissenschaften in November 1915 he continued introducing, then removing, an additional term in his equation. This additional term was invented to assure a steady Universe, and was not needed in the expanding Universe model which held for decades, after Hubble's discovery of the red shift from distant galaxies.

However, recent discoveries of the Universe accelerating and decelerating expansion in cosmological times (several billion years) made this term again necessary. It is small but different from zero. Possible explanations are many, from non-zero energy state of the vacuum; in reality, we completely ignore the physical reason for the presence of this term.

Physics showed many ways to detect the Universe: using the whole spectrum of the electromagnetic radiation, using other elementary particles, like muons, neutrinos, deducing on gravitational waves and so on. However, the exact observations of the past expansion (using some distant star flares, like supernovae and cephaloids) showed that we are experimentally aware of only 4% of the surrounding us "Global

Entity". In other words, all we see, thanks to known to us interactions between matter is only 4% of the Universe. The rest is a dark matter, exceeding the visible matter by three folds, and even more is the dark energy, changing up-and-down the acceleration of the Universe expansion [9].

Coming back to Einstein we note that successive steps in generalizing the Newton law, through the special and then general relativity passed from Euclidean to Minkowski then pseudo-Riemannian metric. Now, we face a situation that the cosmological term is different from zero: maybe a new metric would be needed to describe the Universe? Recent measurements of the cosmic radiation anisotropy [11] showed that the geometry of the Universe is flat (i.e. Euclidean) but more complicated shapes, maybe dodecaheral as shown in the Fig. 3 are not to be excluded.

To complete the question of our ignorance of the general relativity we quote the SXS Internet project from Caltech University [10] "Einstein's equations can be written in a beautifully simple form: $G = 8\pi T$. The G term on the left side represents all the curvature of spacetime at a point, while the T term on the right represents the mass at a point, and its properties. This is the elegant part. The complicated part comes when we realize that this formula is almost completely useless for doing actual calculations. To use it, we have to expand it into at least ten different equations, each with dozens of terms. It is possible to solve the equations with pencil and paper in very special situationswhen most of the dozens of terms happen to be zeroor in situations with low speeds, small masses, and large distanceswhen most of the dozens of terms happen to be very small and practically zero."

6. Concluding remarks

The conclusion is that we have large experimental evidence indicating the need for a revolution in Physics, like it happened in Planck's time one hundred years ago. With high probability, also the Mathematics is ready. What we need is to merge Physics and Mathematics platforms. Therefore we need to discuss jointly the new experimental facts and emerging ideas. In this sense, I thank prof. J. Ławrynowicz and prof. R. Ingarden for the invitation to the Hypercomplex Seminar.

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DOŚWIADCZALNA FIZYKA WSPÓŁCZESNA: DLACZEGO POTRZEBUJEMY NOWEJ MATEMATYKI?

S t r e s z c z e n i e

Sto lat temu wydawało się, że obraz fizyki jest zamknięty i niezbędne są jedynie małe poprawki. Ale wkrótce została odkryta mechanika kwantowa, korzystająca z aparatu matematycznego rozwiniętego w poprzednim wieku.

Wydaje się, że w niezbyt odległej przyszłości czeka nas podobna rewolucja. Wiele wyników fizyki doświadczalnej nie znajduje rozsądnego wyjaśnienia. Nie potrafimy przewidzieć mas kwarków, wyjaśnić działania nadprzewodników wysokotemperaturowych, kompletnym zaskoczeniem było odkrycie ciemnej masy i energii, stanowiących 96% całego Wszechświata, a kompletnie wymykających się naszym metodom obserwacyjnym. Nowe pomysły matematyczne są pilnie potrzebne w fizyce.