

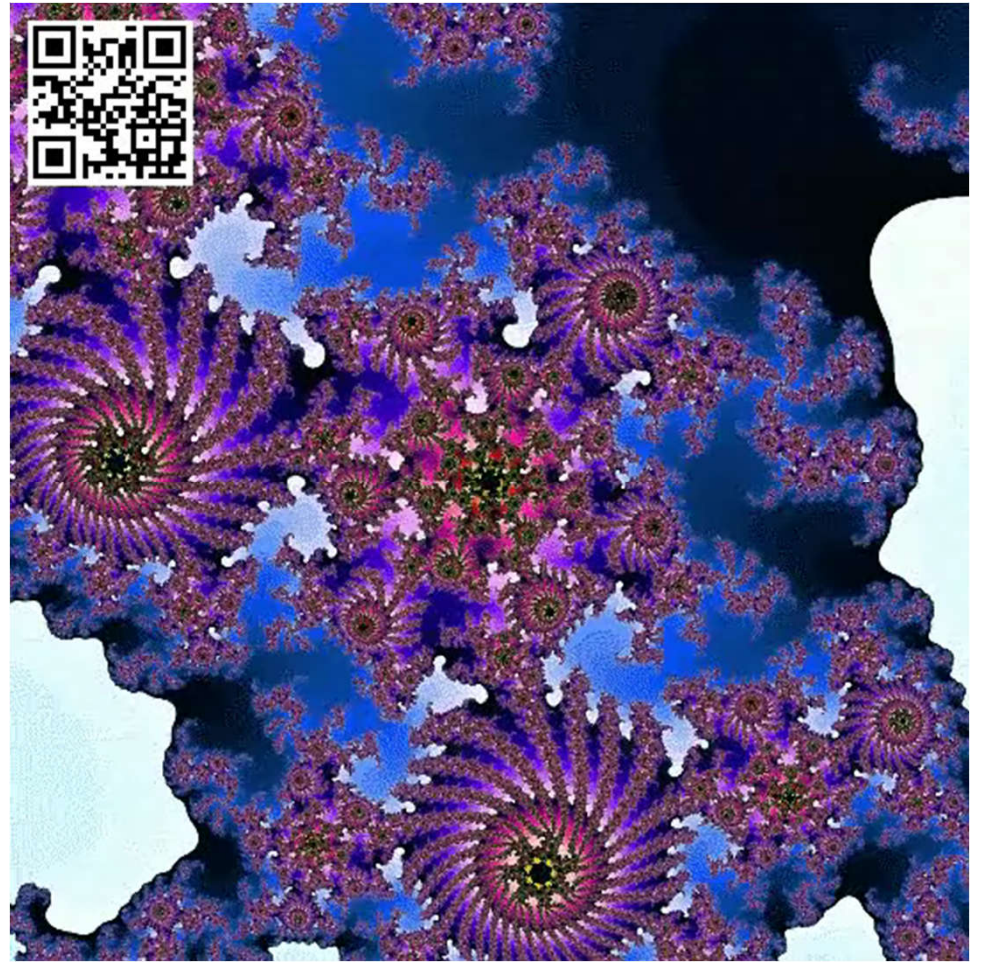
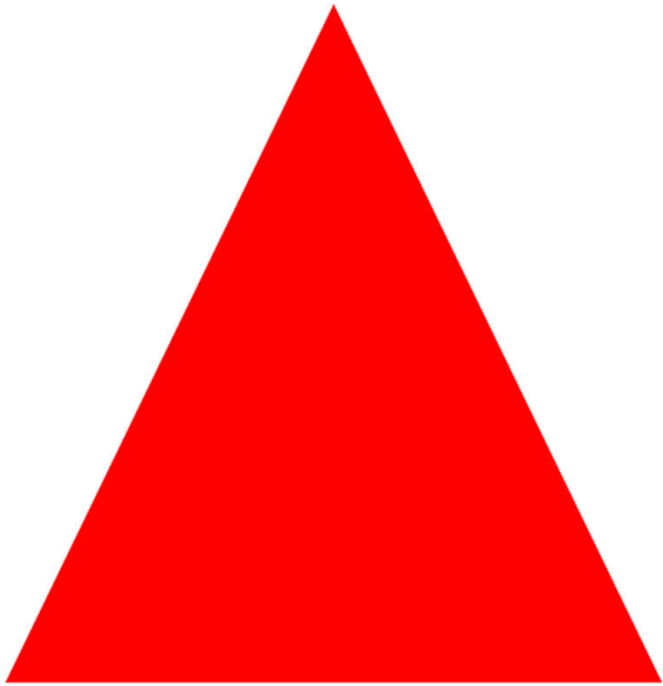
# *Insegnare STEAM con la realtà aumentata*

Grzegorz Karwasz

*Lezione 5: Matematica – regina delle scienze*  
*Parte III: Didattica digitale ed aumentata*

«Inutile perdere il tempo per operazioni che con la mia invenzione qualsiasi contadino potrà fare» (W. Leibniz)

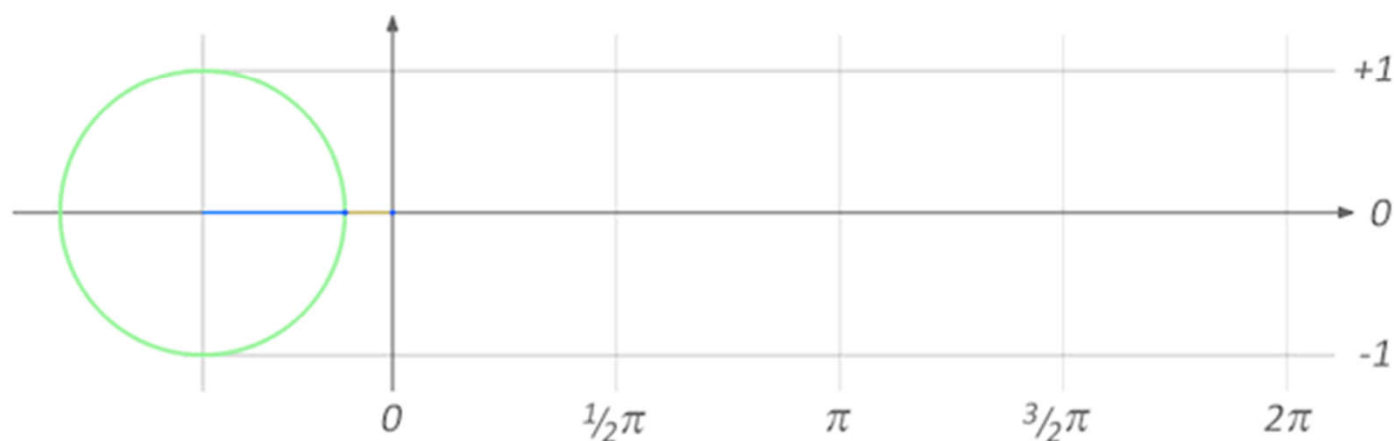
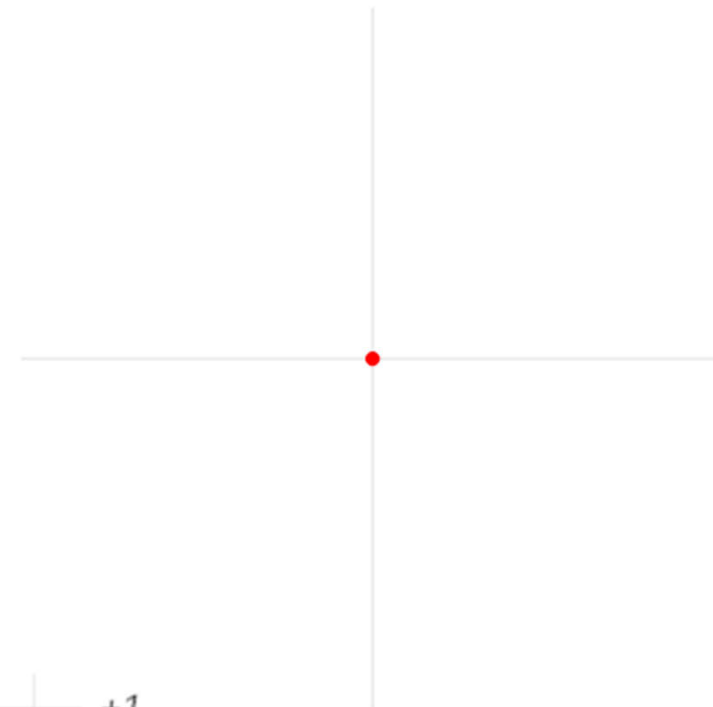
# Ancora i frattali



Il triangolo di Sierpinski

<https://giphy.com/gifs/animation-trippy-psychedelic-xUPGcxdhS5isNwqsuY>

# Ancora il numero pi-greco



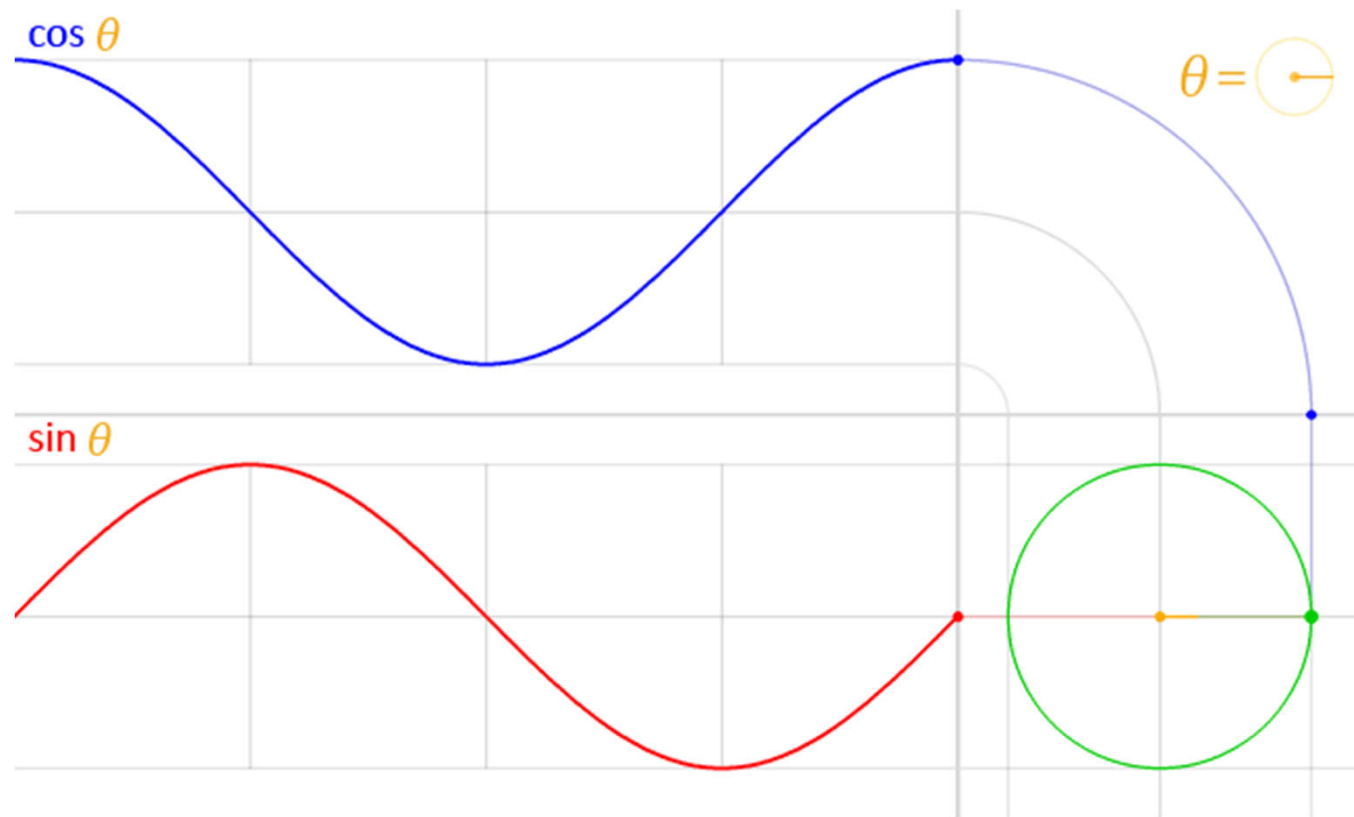
<https://archive.nerdist.com/three-gifs-that-make-pi-instantly-understandable/>

## Due esempi del moto



Il cerchio è la somma di seno e coseno

$$x = \cos \theta$$
$$y = \sin \theta$$



La cicloide (per matematici)  
oppure brachisto-crona (per fisici)

$$\begin{aligned}x &= r(t - \sin t) \\ y &= r(1 - \cos t),\end{aligned}$$



# Brachistocrona (J. Bernoulli)



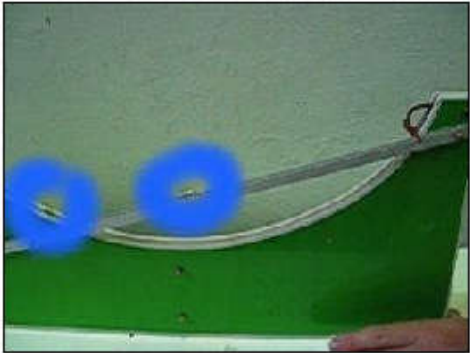
**Meccanica****Ottica****Termodinamica****Elettromagnetismo**

## Scorciatoia

Avendo a disposizione due strade con diversi angoli d'inclinazione e diverse lunghezze, non sappiamo quale scegliere? Dipende se vuoi risparmiare tempo o gambe.

Se hai fretta "conviene" prima prendere velocità su una ripida discesa e poi giungere rapidamente alla meta. Prendere velocità in rettilineo richiede molto più tempo.

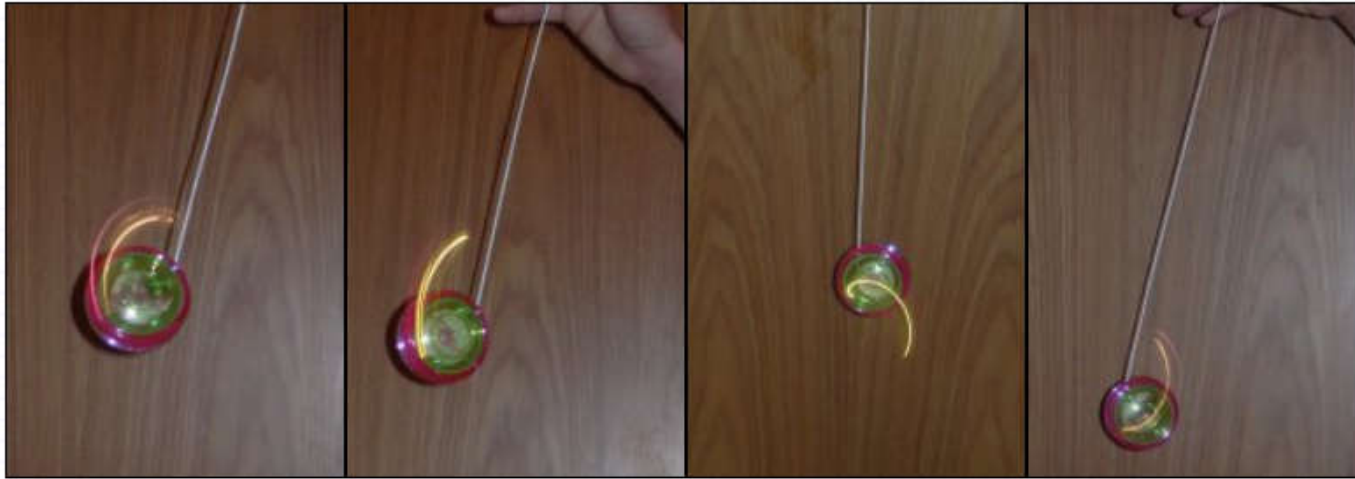
Risulta che non sempre la strada più corta è quella più veloce. A volte, affrettandosi per il treno, vale la pena di prendere la strada più lunga, ma ripida.



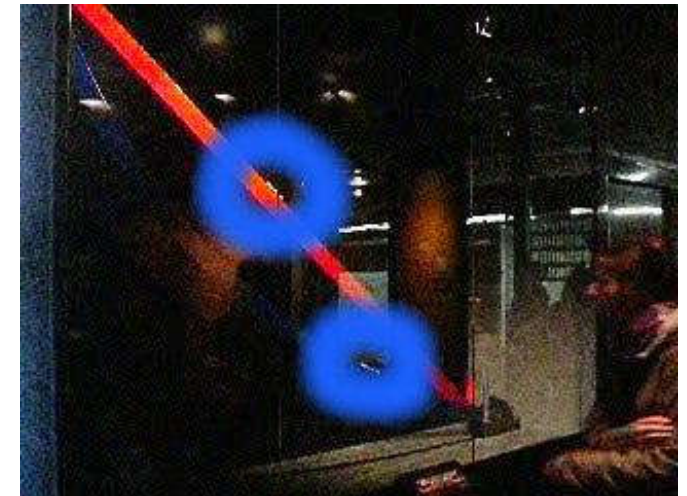
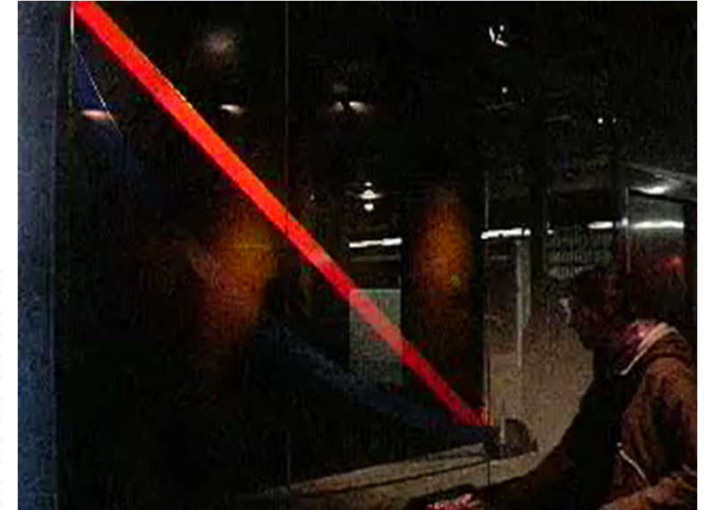
[Più...](#)



# Brachistocrona (J. Bernoulli)



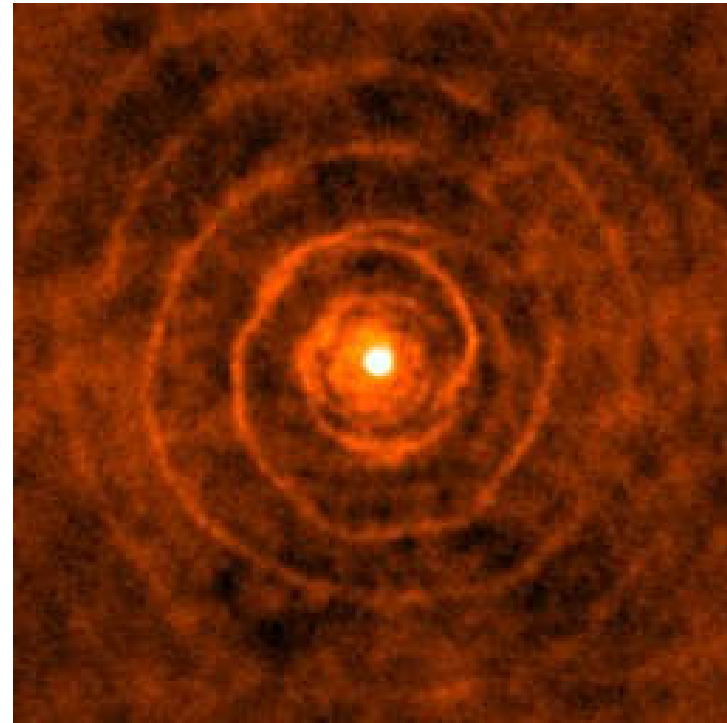
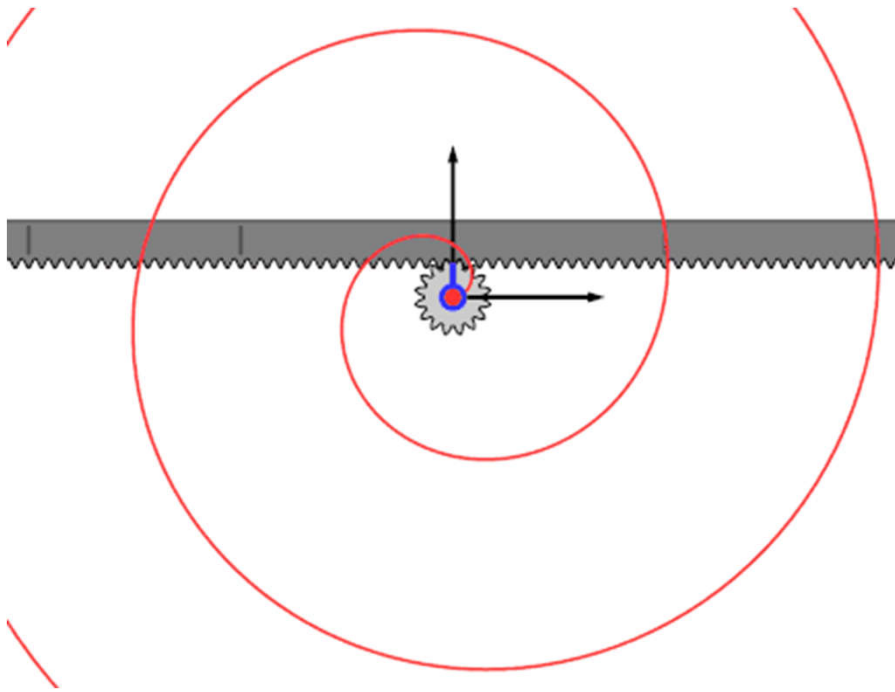
Il problema della caduta più rapida fu posto da un matematico svizzero, Johann Bernoulli, nel 1696 e risolto poco dopo dal fratello, Jakob Bernoulli, che era matematico e fisico: in base alla legge di Bernoulli gli aerei volano e la pallina resta sospesa nella corrente del phon.



<http://dydaktyka.fizyka.umk.pl/zabawki1/files/mech/faster-it.html>

# Spirale di Archimede

$$r = a + b \cdot \theta$$



[https://upload.wikimedia.org/wikipedia/commons/0/00/Archimedean\\_Spiral\\_animation\\_with\\_Gears\\_positive\\_tracing.gif](https://upload.wikimedia.org/wikipedia/commons/0/00/Archimedean_Spiral_animation_with_Gears_positive_tracing.gif)

[https://en.wikipedia.org/wiki/Archimedean\\_spiral](https://en.wikipedia.org/wiki/Archimedean_spiral) Atacama Large Millimeter Array image of [LL Pegasi](#)

# Una spirale più bella

<https://www.geeksforgeeks.org/draw-colorful-spiral-web-using-turtle-graphics-in-python/>

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## Draw Colorful Spiral Web Using Turtle Graphics in Python

Difficulty Level : Basic • Last Updated : 20 Oct, 2020

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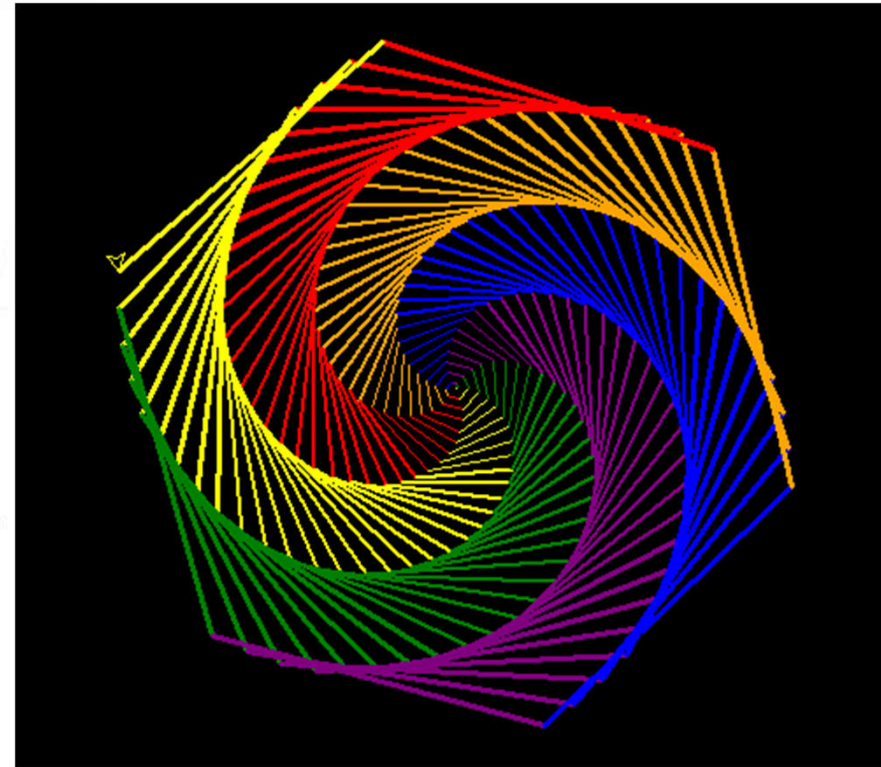


**Prerequisite:** [Turtle Basics](#)

"Turtle" is a Python feature like a drawing board, which lets us command a turtle to draw all over it. This comes packed with the standard Python package and need not be installed externally.

**Methods used:**

- **forward(value):** moves the turtle in the forward direction.
- **turtle.Pen():** setup the turtle pen
- **speed(value):** changes the speed of the turtle
- **width(value):** set the width
- **left(value):** moves the turtle left.



# Math is Fun: pentagono regolare

## Pentagon

A *regular pentagon* has five lines of *reflectional symmetry*, and *rotational symmetry* of order 5 (through  $72^\circ$ ,  $144^\circ$ ,  $216^\circ$  and  $288^\circ$ ). The *diagonals* of a *convex* regular pentagon are in the *golden ratio* to its sides. Given its side length  $t$ , its height  $H$  (distance from one side to the opposite vertex), width  $W$  (distance between two farthest separated points, which equals the diagonal length  $D$ ) and circumradius  $R$  are given by:

$$H = \frac{\sqrt{5 + 2\sqrt{5}}}{2} t \approx 1.539 t,$$

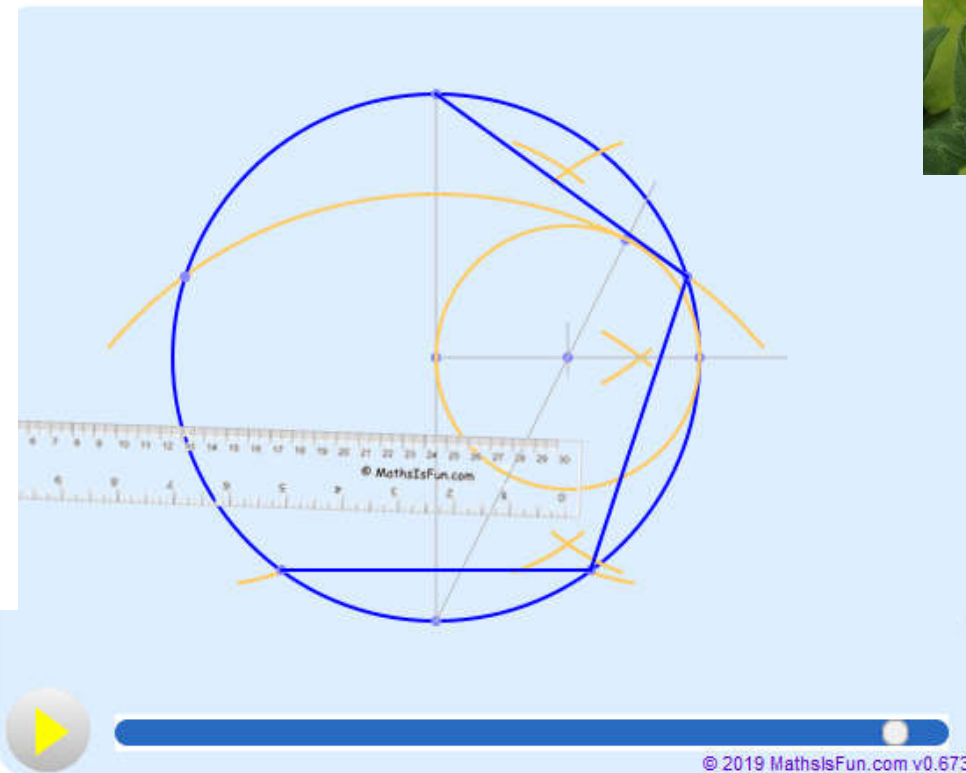
$$W = D = \frac{1 + \sqrt{5}}{2} t \approx 1.618 t,$$

$$W = \sqrt{2 - \frac{2}{\sqrt{5}}} \cdot H \approx 1.051 H,$$

$$R = \sqrt{\frac{5 + \sqrt{5}}{10}} t \approx 0.8507 t,$$

$$D = R \sqrt{\frac{5 + \sqrt{5}}{2}} = 2R \cos 18^\circ = 2R \cos \frac{\pi}{10} \approx 1.902 R.$$

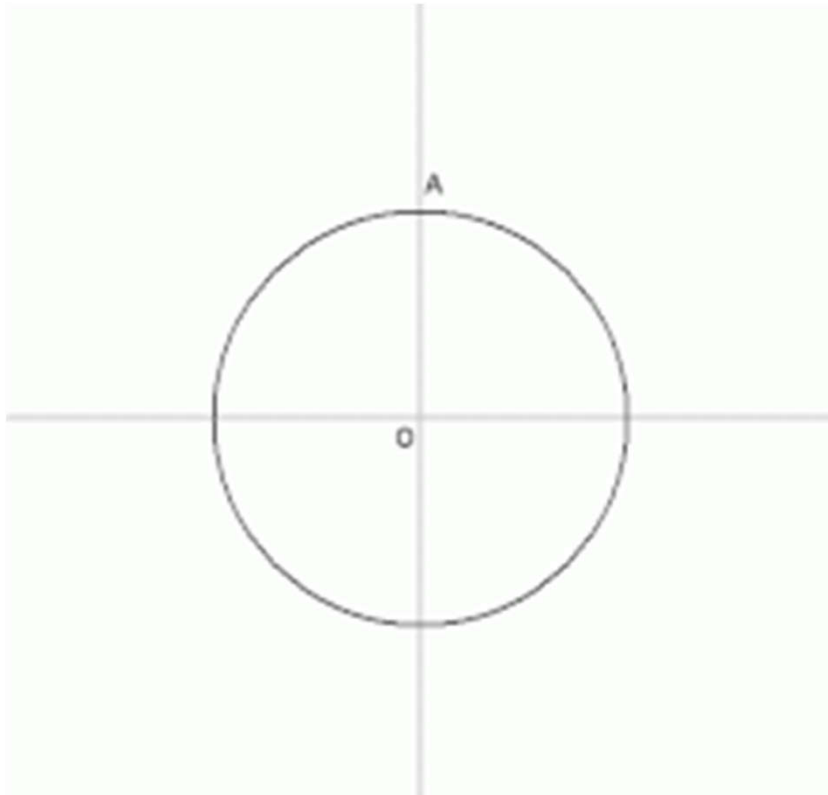
How to construct a [Regular Pentagon](#) using just a compass and a straightedge



<https://en.wikipedia.org/wiki/Pentagon>

<https://www.mathsisfun.com/geometry/construct-pentagon.html>

# Pentagono regolare: due altre costruzioni



[https://upload.wikimedia.org/wikipedia/commons/thumb/9/96/Regular\\_Pentagon\\_Using\\_Carlyle\\_Circle.gif/220px-Regular\\_Pentagon\\_Using\\_Carlyle\\_Circle.gif](https://upload.wikimedia.org/wikipedia/commons/thumb/9/96/Regular_Pentagon_Using_Carlyle_Circle.gif/220px-Regular_Pentagon_Using_Carlyle_Circle.gif)

$$D = \frac{1 + \sqrt{5}}{2} t$$

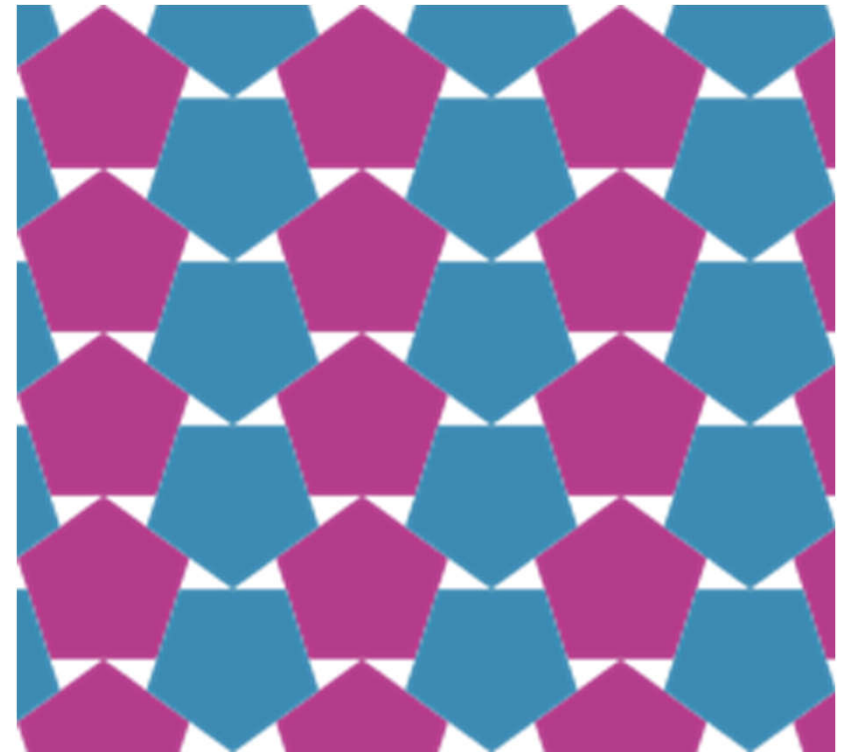


[https://en.wikipedia.org/wiki/Pentagon#/media/File:01-Pentagon-Euklid\\_Animation.gif](https://en.wikipedia.org/wiki/Pentagon#/media/File:01-Pentagon-Euklid_Animation.gif)

# Il problema del piastrellista

A regular pentagon cannot appear in any tiling of regular polygons. First, to prove a pentagon cannot form a [regular tiling](#) (one in which all faces are congruent, thus requiring that all the polygons be pentagons), observe that  $360^\circ / 108^\circ = 3\frac{1}{3}$  (where  $108^\circ$  is the interior angle), which is not a whole number; hence there exists no integer number of pentagons sharing a single vertex and leaving no gaps between them. More difficult is proving a pentagon cannot be in any edge-to-edge tiling made by regular polygons:

The maximum known [packing density](#) of a regular pentagon is approximately 0.921, achieved by the [double lattice](#) packing shown. In a preprint released in 2016, [Thomas Hales](#) and Wöden Kusner announced a proof that the double lattice packing of the regular pentagon (which they call the "pentagonal ice-ray" packing, and which they trace to the work of Chinese artisans in 1900) has the optimal density among all packings of regular pentagons in the plane.<sup>[\[15\]](#)</sup> As of 2020, their proof has not yet been refereed and published.



# Il pentagono in natura

## Plants [\[ edit \]](#)



Pentagonal cross-section of [okra](#).



[Morning glories](#), like many other flowers, have a pentagonal shape.

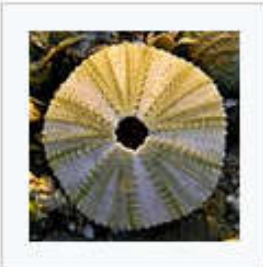


The gynoecium of an [apple](#) contains five carpels, arranged in a [five-pointed star](#)

## Animals [\[ edit \]](#)



A [sea star](#). Many [echinoderms](#) have fivefold radial symmetry.

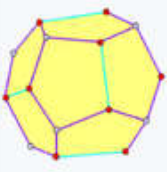
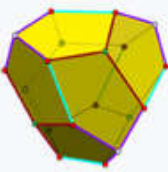
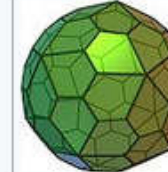
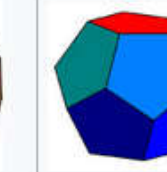


Another example of echinoderm, a [sea urchin](#) endoskeleton.

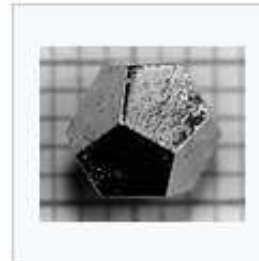


An illustration of [brittle stars](#), also echinoderms with a pentagonal shape.

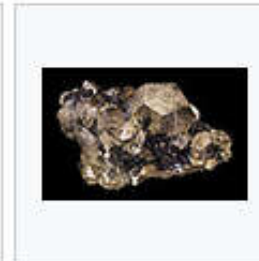
## Pentagons in polyhedra [\[ edit \]](#)

| $I_h$                                                                              | $T_h$                                                                               | $T_d$                                                                               | O                                                                                   | I                                                                                   | $D_{5d}$                                                                            |
|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|  |  |  |  |  |  |
| Dodecahedron                                                                       | Pyritohedron                                                                        | Tetartoid                                                                           | Pentagonal icositetrahedron                                                         | Pentagonal hexecontahedron                                                          | Truncated trapezohedron                                                             |

## Minerals [\[ edit \]](#)



A Ho-Mg-Zn icosahedral [quasicrystal](#) formed as a pentagonal [dodecahedron](#). The faces are true regular pentagons.



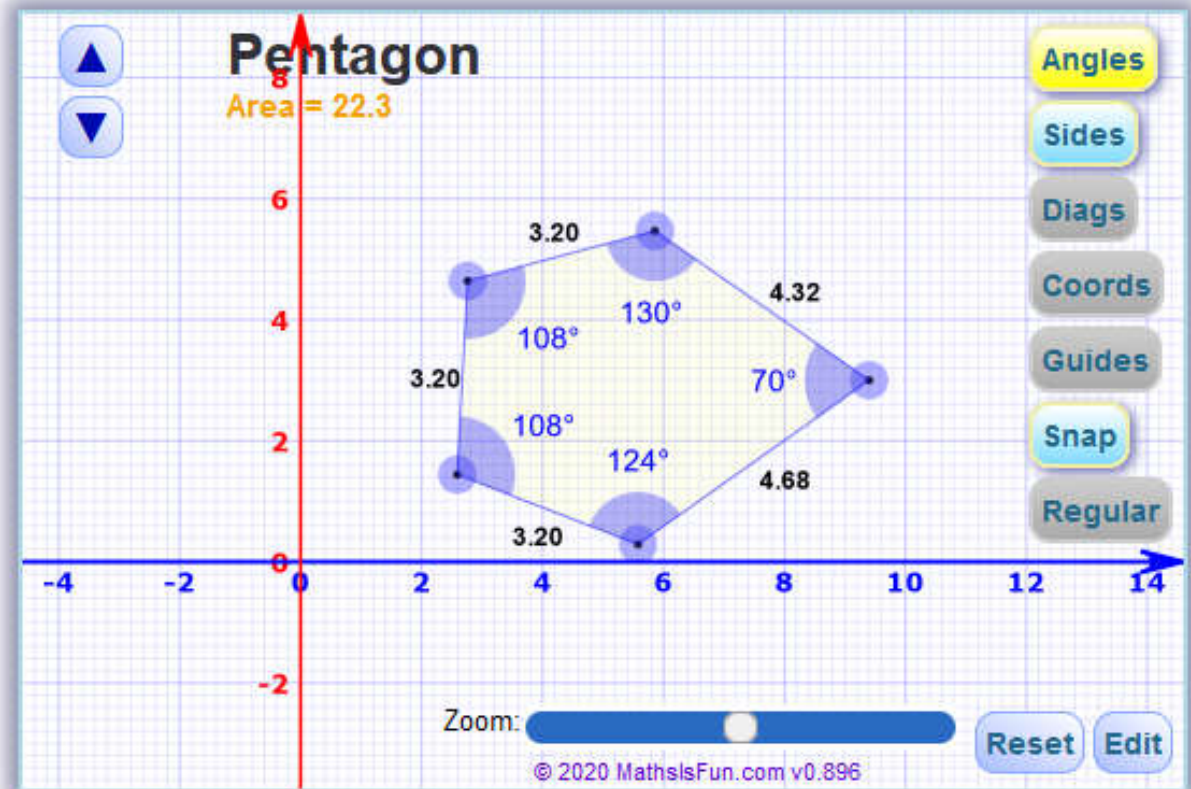
A [pyritohedral](#) crystal of [pyrite](#). A pyritohedron has 12 identical pentagonal faces that are not constrained to be regular.

<https://en.wikipedia.org/wiki/Pentagon>

# Math is Fun

Can you draw your polygon? Maybe you know the coordinates, or lengths and angles, either way this can give you a good **estimate** of the Area.

*Note: due to computer rounding errors the last digit is not always correct.*



[Edit](#)

<https://www.mathsisfun.com/geometry/area-polygon-drawing.html>

Use the "Edit" button to manually edit the coordinates, or to **enter new coordinates of your own**

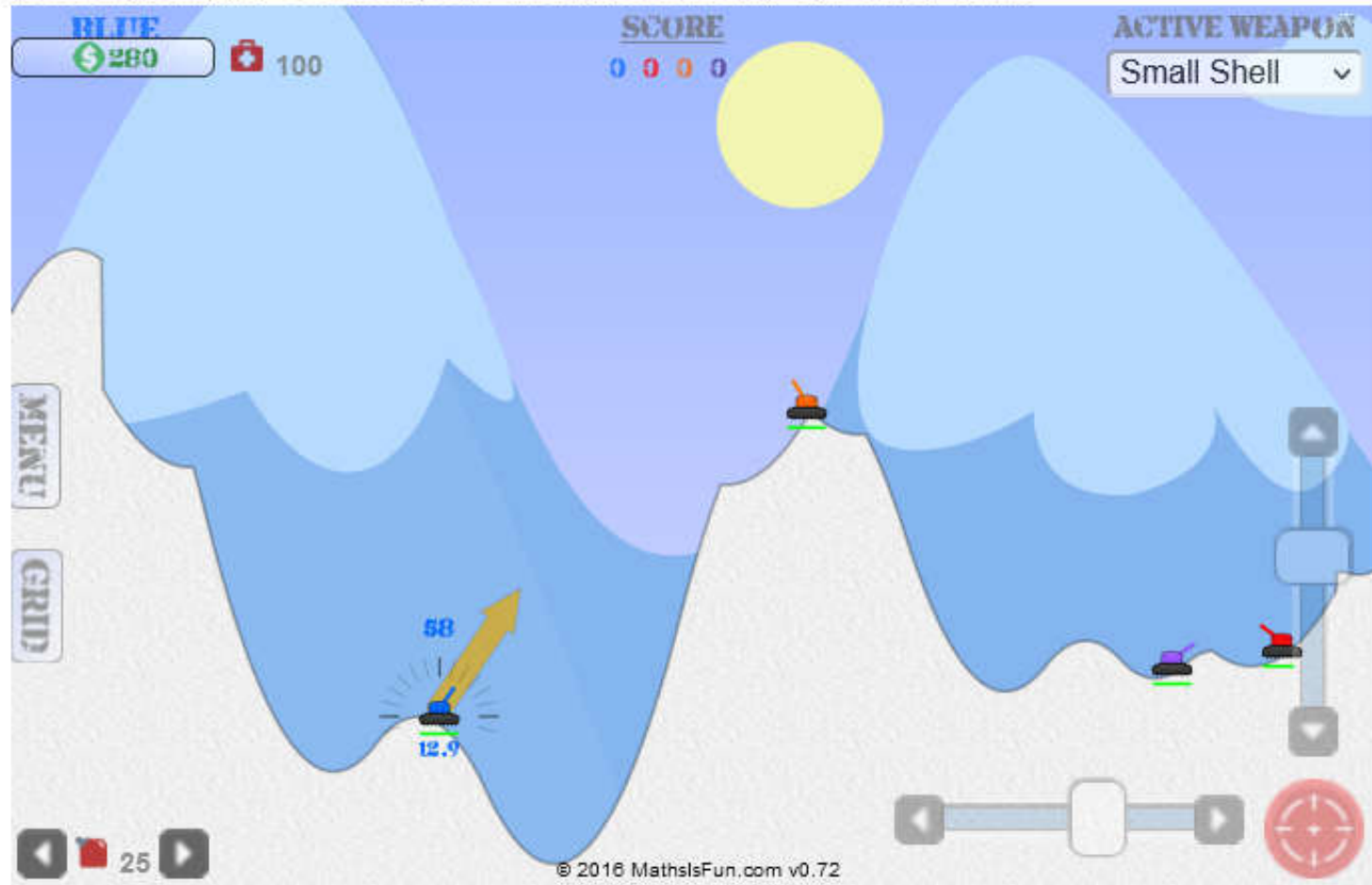
# Math is Fun



<https://www.mathsisfun.com/>

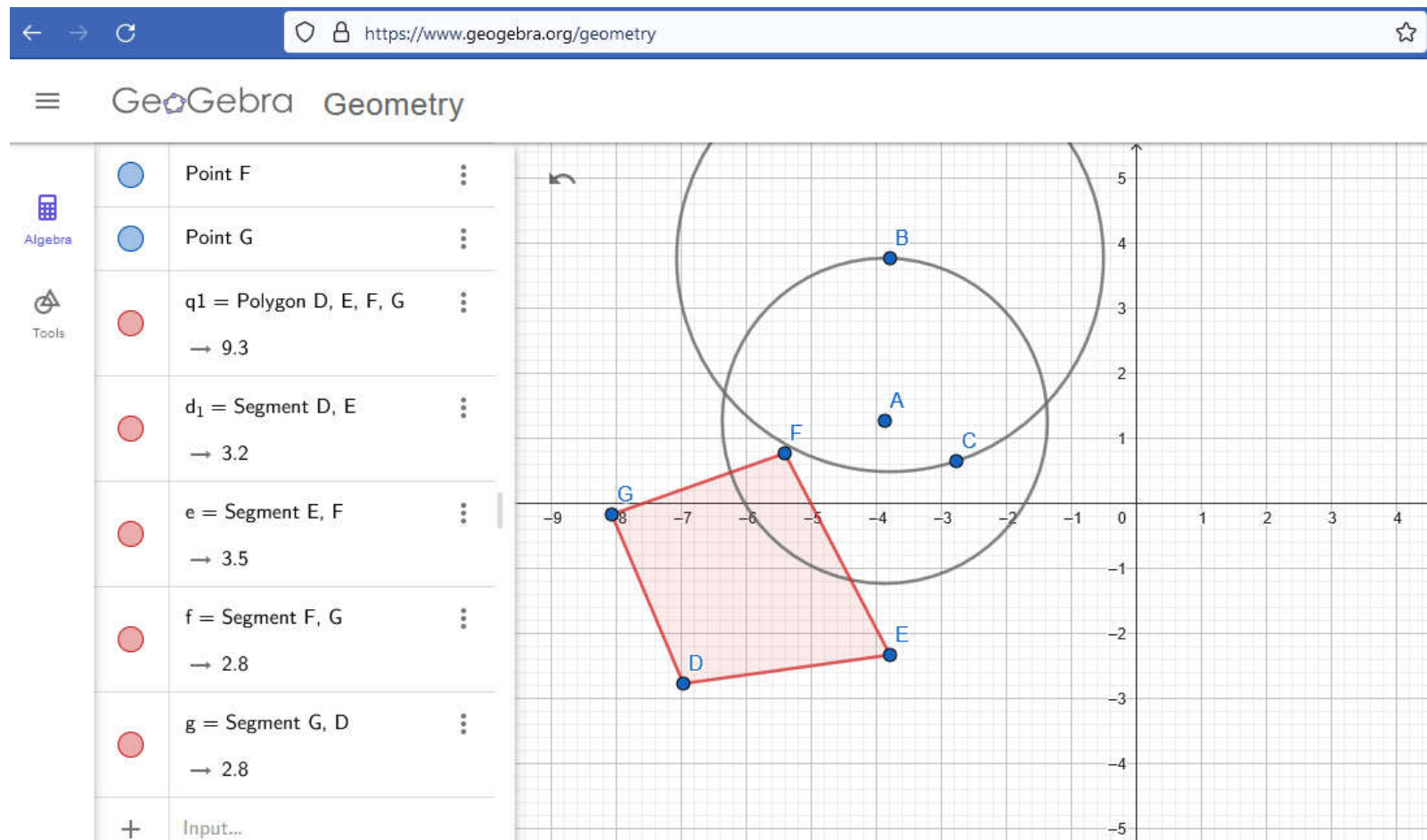
# Math is Fun: la battaglia di carri armati

Adjust force and angle. Buy weapons and fuel. Destroy the other tank.

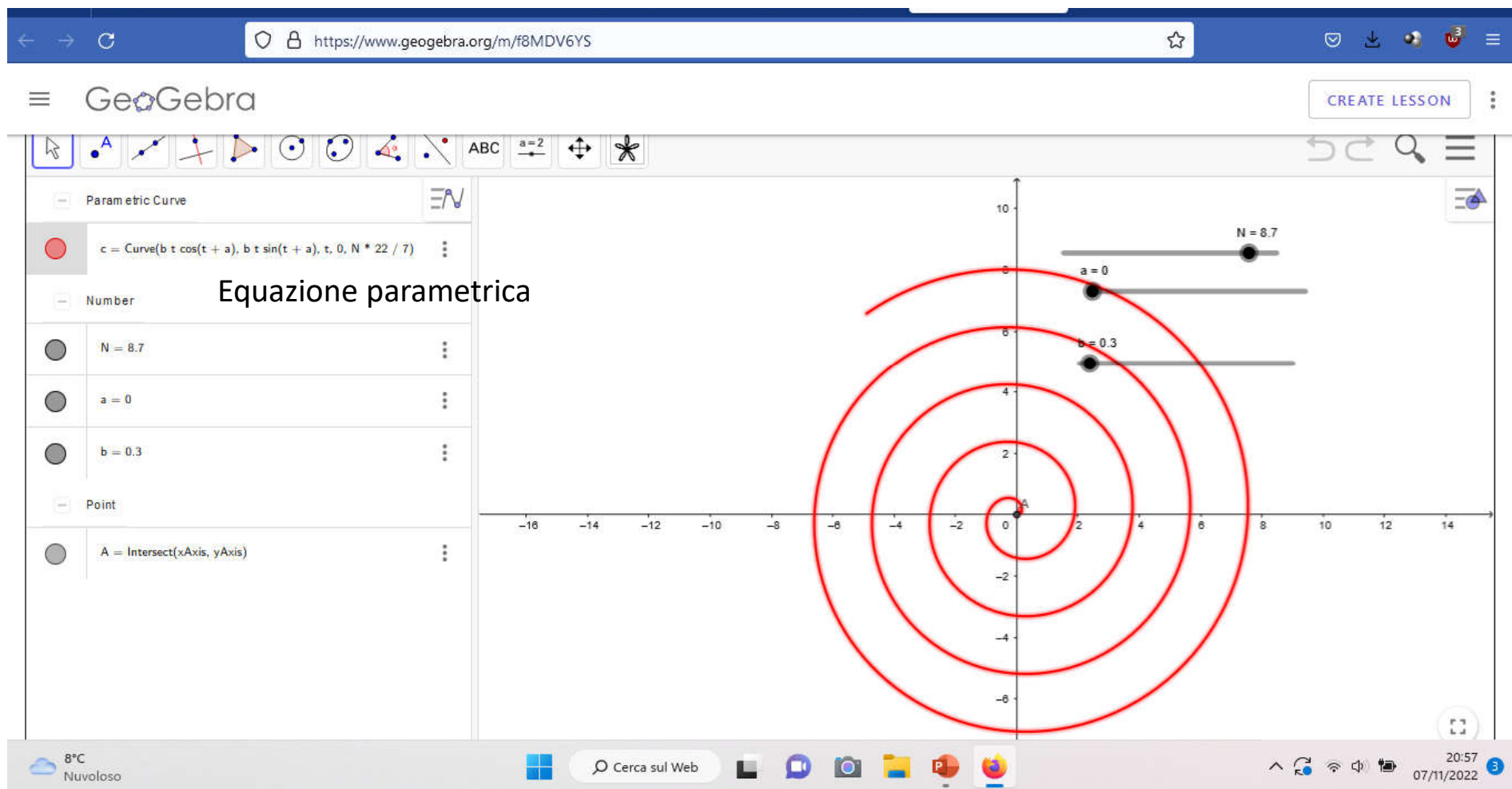


<https://www.mathsisfun.com/games/tanks2.html>

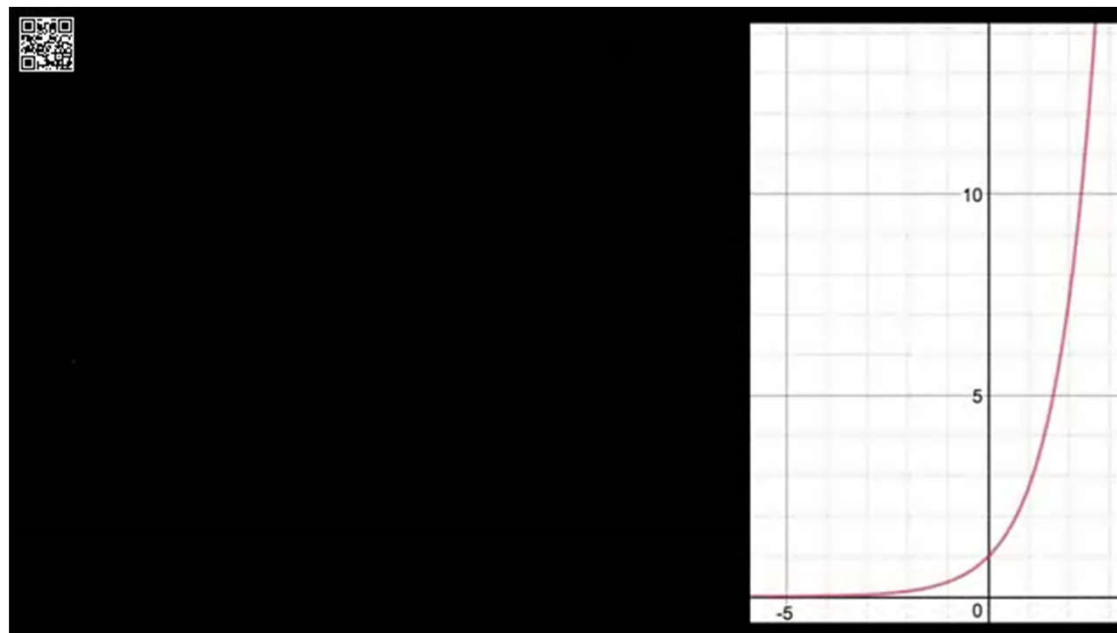
# Lo strumento didattico serio: Geogebra



# Lo strumento didattico serio: Geogebra



# Khan's Academy



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📄 Proving the derivatives of  $\sin(x)$  and  $\cos(x)$

▶ Derivative of  $e^x$

▶ Derivative of  $\ln(x)$

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Derivatives of  $e^x$  and  $\ln(x)$

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$$\frac{d}{dx}[7\cos(x) + 3e^x] = 7\sin(x) + 3e^x$$

Stuck? Review related a

|            |               |       |               |
|------------|---------------|-------|---------------|
| +          | -             | ·     | $\frac{x}{y}$ |
| sin        | cos           | tan   | $\theta$      |
| $\sqrt{x}$ | $\sqrt[3]{x}$ | $a^b$ | $\pi$         |

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# Derivatives of $e^x$ and $\ln(x)$

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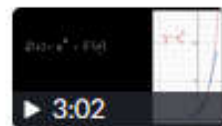
$$\frac{d}{dx}[7\cos(x) + 3e^x] = \boxed{-7\sin(x) + 3e^x}$$

1 / 3

Recall that  $\frac{d}{dx}[e^x] = e^x$  and  $\frac{d}{dx}[\cos(x)] = -\sin(x)$ .

[+ Get another hint \(1/3\)](#)

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- Derivative of  $e^x$
- Derivative of  $\ln(x)$
- Practice: Derivatives of  $e^x$  and  $\ln(x)$
- Proof: The derivative of  $e^x$  is  $e^x$
- Proof: the derivative of  $\ln(x)$  is  $1/x$

Next lesson  
The product rule

# Derivatives of $e^x$ and $\ln(x)$

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$$\frac{d}{dx}[-\ln(x) + x^2] = -\frac{1}{x} + 2x$$

1 / 3 Recall that  $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$  and  $\frac{d}{dx}[x^n] = nx^{n-1}$ .

2 / 3

$$\begin{aligned} \frac{d}{dx}[-\ln(x) + x^2] \\ &= -1 \cdot \frac{d}{dx}[\ln(x)] + \frac{d}{dx}[x^2] \\ &= -1 \cdot \frac{1}{x} + (2x^1) \\ &= -\frac{1}{x} + 2x \end{aligned}$$

3 / 3 In conclusion,

$$\frac{d}{dx}[-\ln(x) + x^2] = -\frac{1}{x} + 2x$$

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Proof: The derivative of  $e^x$  is  $e^x$

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 Proving the derivatives of  $\sin(x)$  and  $\cos(x)$

 **Derivative of  $e^x$**

 Derivative of  $\ln(x)$

 Practice: Derivatives of  $e^x$  and  $\ln(x)$

 **Proof: The derivative of  $e^x$  is  $e^x$**

 Proof: the derivative of  $\ln(x)$  is  $1/x$

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# Proof: The derivative of $e^x$ is $e^x$

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$e^x$  is the only function that is the derivative of itself!

$$\frac{d}{dx}[e^x] = e^x$$

(Well, actually,  $f(x) = 0$  is also the derivative of itself, but it's not a very interesting function...)

The AP Calculus course doesn't require knowing the proof of this fact, but we believe that as long as a proof is accessible, there's always something to learn from it. In general, it's always good to require some kind of proof or justification for the theorems you learn.

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{n \rightarrow 0} (1+n)^{\frac{1}{n}}$$

$$\frac{d}{dx}[e^x] = e^x$$

$$= \lim_{\Delta x \rightarrow 0} \frac{e^{x+\Delta x} - e^x}{\Delta x}$$

$$= e^x \lim_{\Delta x \rightarrow 0} \frac{e^{\Delta x} - 1}{\Delta x}$$

$$= e^x \lim_{n \rightarrow 0} \frac{\frac{1}{n} \ln(n+1)}{1}$$

$$\frac{d}{dx}[e^x] = \lim_{\Delta x \rightarrow 0} \frac{e^{x+\Delta x} - e^x}{\Delta x}$$

$$= e^x \lim_{\Delta x \rightarrow 0} \frac{e^{\Delta x} - 1}{\Delta x}$$

$$= e^x \lim_{n \rightarrow 0} \frac{\frac{1}{n} \ln(n+1)}{1}$$

$$n+1 = e^{\Delta x}$$

$$\ln(n+1) = \Delta x$$

$$\Delta x \rightarrow 0 \Rightarrow n \rightarrow 0$$

$$y = e^x$$

$$a \ln b = \ln(b^a)$$

from desmos.com

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# Anche bambini



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# Khan Academy for Kids



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2 Hz + 3 Hz

Time

2.00 cycles/second

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0:12 / 20:56

The Fourier transform.

x-coordinate of center of mass

Frequency

Uncertainty principle

Quantum 19:21

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(2+1) · (1+3)

= 2 · 7 + 3

2 7 3

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factor  $2x^5 - 19x^4 + 58x^3 - 67x^2 + 56x - 48$

**NATURAL LANGUAGE** **MATH INPUT** **EXTENDED KEYBOARD** **EXAMPLES** **UPLOAD** **RANDOM**

Input interpretation

factor  $2x^5 - 19x^4 + 58x^3 - 67x^2 + 56x - 48$

Result ☒ Step-by-step solution

$(2x - 3)(x - 4)^2(x^2 + 1)$

**Wolfram|Alpha Step-by-step solution**

Results: **Hide hints**

**STEP 1**

Factor the following:  
 $2x^5 - 19x^4 + 58x^3 - 67x^2 + 56x - 48$

**STEP 2**

Hint: Find all linear factors of  $2x^5 - 19x^4 + 58x^3 - 67x^2 + 56x - 48$  via the rational root theorem. Do this by finding rational roots. The candidates are  $x = \pm \frac{p}{q}$  for

Scomposizione dei polinomi  
Suggerimenti passo per passo

per es. «cerchiamo i divisori del 48»

# Anche per Fisica «abbiamo» creato un sito simile

## Collection of Solved Problems in Physics



MechanicsThermodynamicsElectricity and magnetismOpticsSmall-scale physics

About

Show task

code:

Physics — Contents

- Mechanics
- Thermodynamics
- Electricity and magnetism
- Optics
- Small-scale physics

Search

## Welcome in Collection of Solved Problems in Physics

This collection of Solved Problems in Physics is developed by [Department of Physics Education](#), [Faculty of Mathematics and Physics](#), [Charles University in Prague](#) since 2006.

The Collection contains tasks at various level in mechanics, electromagnetism, thermodynamics and optics. The tasks mostly do not contain the calculus (derivations and integrals), on the other hand they are not simple such as the use just one formula. Tasks have very detailed solution descriptions. **Try to solve the task or at least some of its parts independently.** You can use hints and task analysis (solution strategy).

Level and category of the task is indicated by an icon in the right upper corner.

Difficulty level

- L1** Level 1 – Lower secondary level
- L2** Level 2 – Upper secondary level
- L3** Level 3 – Advanced upper secondary level
- L4** Level 4 – Undergraduate level

-  Qualitative task
-  Graphical task
-  Task with unusual solution

<http://physicstasks.eu/en>

## Mechanics

### Mechanics

### Thermodynamics

### Electricity and magnetism

### Optics

### Small-scale physics

### About

### Show task

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### Tasks

### Task filter »

#### • Kinematics of mass point (30)

- Motion Given by a Motion Graph I (L2)
- Motion Given by a Motion Graph II (L3)
- Motion Given by a Motion Graph III (L4)
- A lift (L2)
- Annie's Ride (L1)
- Passing of a train I (L1)
- Passing of a train II (L1)
- Moving Boat (L2)
- Rescue Plane (L2)
- Rolling of a Ball (L2)
- Basketball player (L2)
- A garden hose (L3)
- Water streaming out of the tank (L3)
- A Cannon Firing Down a Hill (L4)
- Cottage Dwellers (L2)
- The Voyage of a Raft (L3)
- Mouse and Cat (L3)
- Sliding of a Line Segment I (L3)
- An ant on a rod (L4)
- A ladybug crawling on a rotating cylinder (L4)
- Movement of a Particle I (L4)

## Rescue Plane

Task number: 1992

L2

RU

<a>

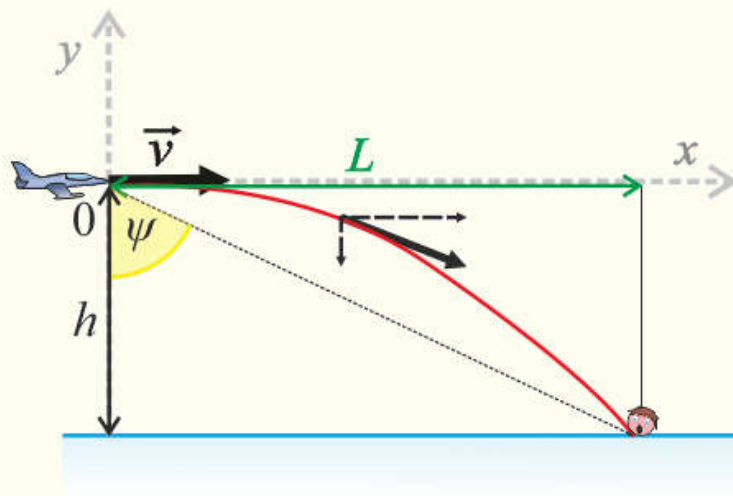
🇨🇪  
🇸🇰

✉

A rescue plane flies to help a drowning man. The pilot keeps the plane at a constant height of 1200 m above the water surface and heads directly above the man's head (see the diagram). The speed of the plane is  $430 \text{ km} \cdot \text{h}^{-1}$ .

At what angle of view must the pilot release the rescue bag in order to let it fall as close to the man as possible?

Neglect the resistance of environment.



<http://physicstasks.eu/>

- Cylinder (L4)
  - Movement of a Particle I (L4)
  - Movement of a Particle II (L4)
  - Motion of a Drop (L4)
  - Bullet in Vacuum (L4)
  - A Moving Wheel (L4)
  - Angle grinder disc (L4)
  - Wheel Rotation (L3)
  - Little Paulie (L4)
  - Movement of centre of mass of a mass point system (L4)
  - Steel ball (L2)
  - Dynamics of mass point (25)
  - Momentum, work, energy and power (21)
  - Mechanics of rigid body (27)
  - Mechanics of continuum (8)
  - Gravity (8)
- Search

### Given values

|                                          |                                             |
|------------------------------------------|---------------------------------------------|
| $h = 1200 \text{ m}$                     | height of the plane above the water surface |
| $v = 430 \text{ km} \cdot \text{h}^{-1}$ | speed of the plane                          |
| $\psi = ? (^{\circ})$                    | angle of view                               |
| From the tables:                         |                                             |
| $g = 9.81 \text{ m} \cdot \text{s}^{-2}$ | gravitational acceleration                  |

### Hint 1: Initial velocity of the bag

What is the initial velocity of the bag? Do you know its magnitude?

#### Solution of Hint 1

The initial velocity of the bag is identical to the velocity of the plane. Therefore it has the magnitude  $v$ .

### Hint 2: Duration of the fall on the surface

What is the motion of the bag in a vertical direction?

How can you determine the time  $t$  it takes before the bag hits the water surface if you know at what height  $h$  above the surface it was released?

#### Solution of Hint 2

If we neglect the resistance of environment, the bag moves vertically in a free fall.

Because we know at what height it was released, we can easily determine the duration of its fall to the surface. It holds in the direction of the  $y$  axis that:

$$-h = -\frac{1}{2}gt^2,$$

therefore:

wolfram.com

geometria

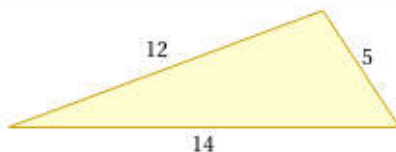
Assuming "triangle" is a geometric object | Use as a [unit](#) instead

Input interpretation

triangle edge lengths 5 | 12 | 14

Visual representation

<https://www.wolframalpha.com/input?i=5%2C+12%2C+14+triangle>



Triangle shape

obtuse triangle

Properties

[Less](#)

[Step-by-step solution](#)

|              |                                         |
|--------------|-----------------------------------------|
| circumradius | $\frac{40}{\sqrt{31}} \approx 7.18421$  |
| inradius     | $\frac{21}{2\sqrt{31}} \approx 1.88586$ |
| area         | $\frac{21\sqrt{31}}{4} \approx 29.2308$ |
| perimeter    | 31                                      |

wolframalpha:

Persino  
la geografia



<https://www.wolframalpha.com/input?i=France>

[Satellite image »](#)

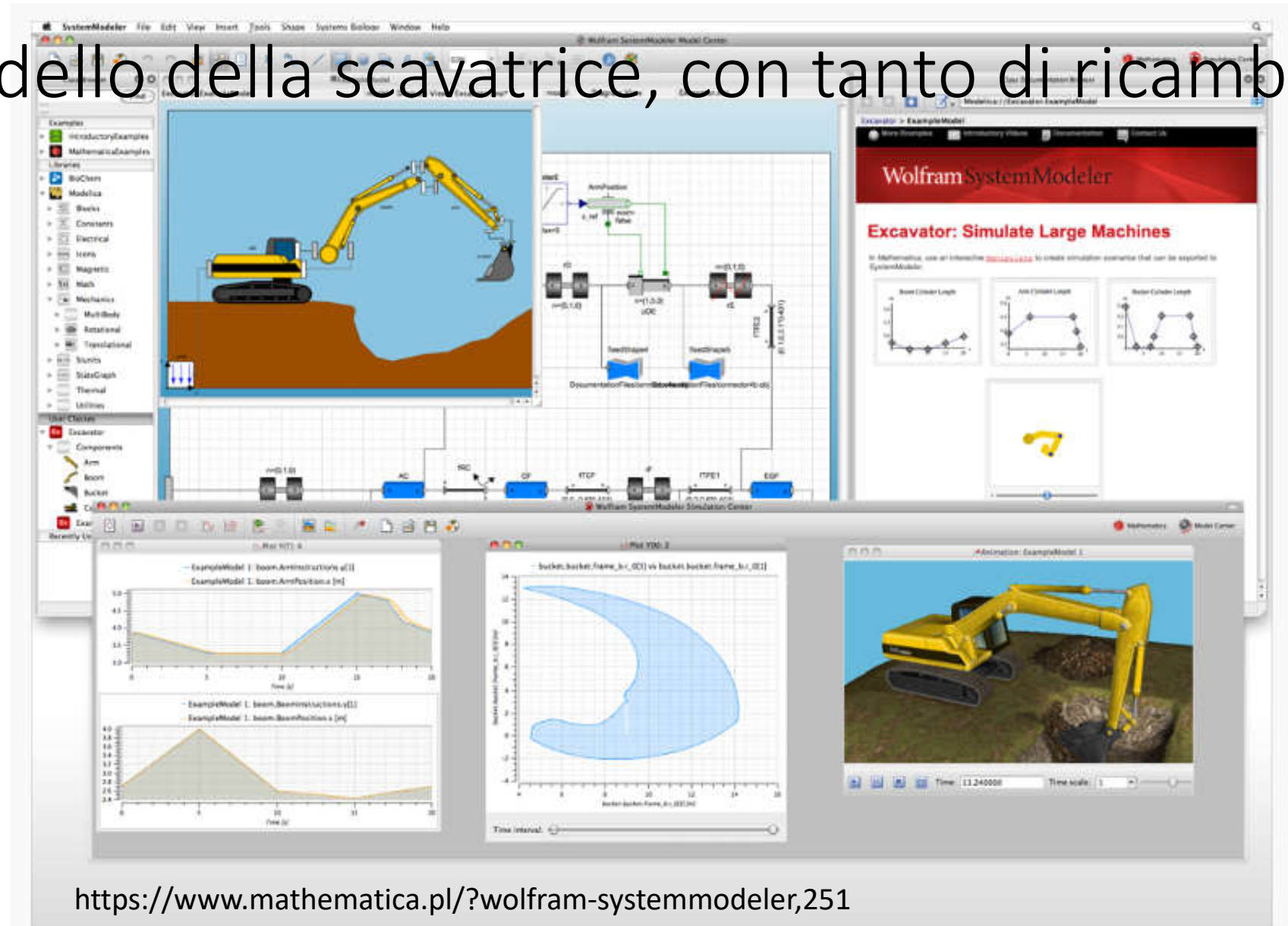
#### Geographic properties

[Enlarge](#) [Data](#) [Customize](#) [Sources](#)

|            |                                                                             |
|------------|-----------------------------------------------------------------------------|
| total area | 551 500 km <sup>2</sup> (square kilometers) (world rank: 49 <sup>th</sup> ) |
| land area  | 549 970 km <sup>2</sup> (square kilometers) (world rank: 49 <sup>th</sup> ) |
| continent  | Europe                                                                      |

|                           |                                                                                                                                                                       |
|---------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| annual-crop land fraction | 33.1% (world rank: 25 <sup>th</sup> )                                                                                                                                 |
| natural resources         | antimony   arsenic   bauxite   coal   feldspar   fish   fluorspar   gypsum   iron   potash   timber   uranium   zinc   petroleum   kaolin   niobium   tantalum   clay |
| coastline length          | 3427 km (kilometers) (world rank: 36 <sup>th</sup> )                                                                                                                  |
| highest point             | 4808 meters (Mont Blanc (mountain))                                                                                                                                   |

# Modello della scavatrice, con tanto di ricambi



# On-line equation solver

WolframAlpha

## Online Equation Solver

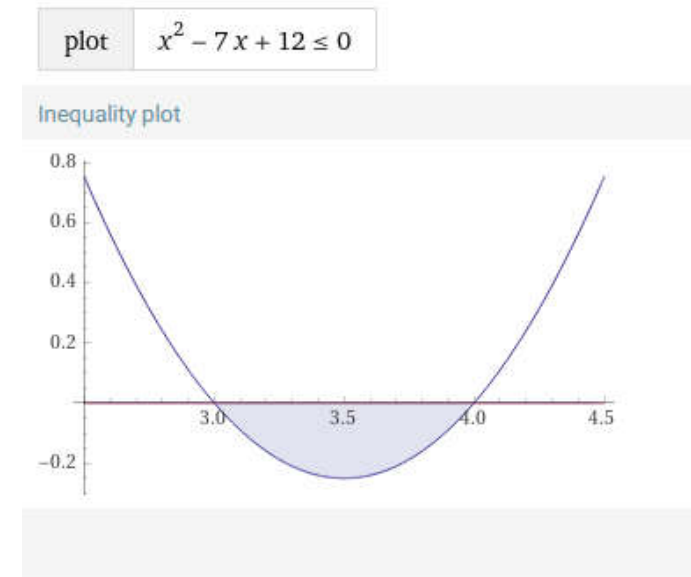
Solve linear, quadratic and polynomial systems of equations with Wolfram|Alpha

plot inequality  $x^2 - 7x + 12 \leq 0$

NATURAL LANGUAGE MATH INPUT

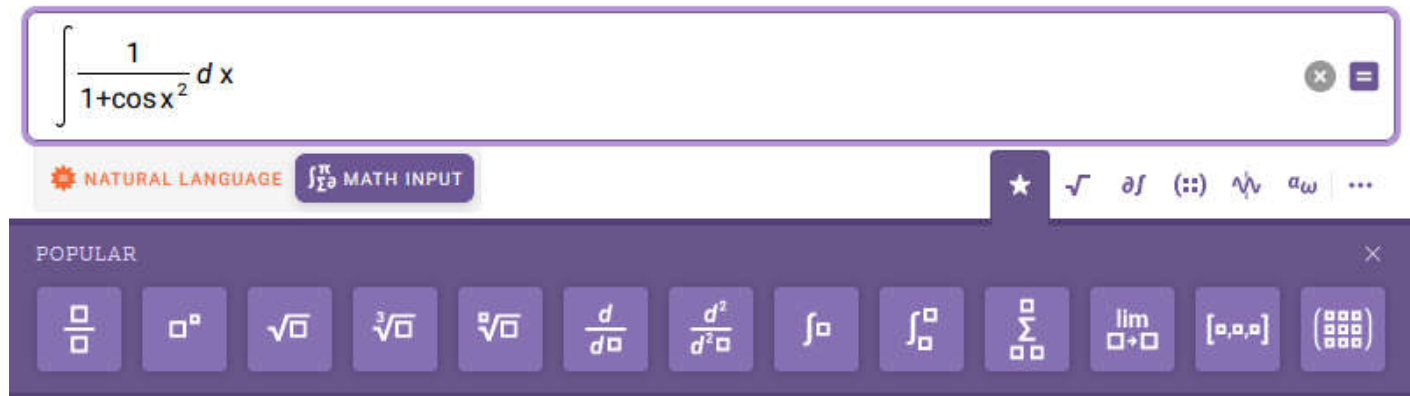
BASIC MATH

Buttons:  $\frac{\square}{\square}$ ,  $\square^2$ ,  $\square^a$ ,  $\sqrt{\square}$ ,  $\sqrt[3]{\square}$ ,  $\sqrt[n]{\square}$ ,  $\infty$ ,  $-\infty$ ,  $\pi$ ,  $e$ ,  $e^{\square}$ ,  $\ln(\square)$ ,  $\log_a(\square)$ ,  $\log_{10}(\square)$ ,  $|\square|$ ,  $\square \leq \square$ ,  $\square \geq \square$ ,  $\square \neq \square$



<https://www.wolframalpha.com/calculators/equation-solver-calculator>

# On-line equation solver



Input

$$\int \frac{1}{1+\cos(x^2)} dx$$

Indefinite integral

(no result found in terms of standard mathematical functions)

Series expansion of the integral at x=0 [More terms](#)

$$\frac{x}{2} + \frac{x^5}{40} + \frac{x^9}{432} + O(x^{13}) + \text{constant}$$

(Taylor series)

$$\int \frac{1}{1+\cos^2(x)} dx = \frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}} + \text{constant}$$

POWERED BY THE WOLFRAM LANGUAGE

Tra poco basterà un conto su wolfram.com per passare la maturità...

# Il riassunto

- La matematica è, senza dubbio, utile, ma può essere in ugual modo *dilettevole*
- Un arduo compito (per l'insegnante, il ministero che definisce cv, gli editori) di trovare un giusto equilibrio, che unisca la didattica, il divertimento, e la scienza.

Grazie per la vostra attenzione!

